

# Gravito-Magnetism in one-body and two-body systems: Theory and Experiments.

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**Summary.** — We survey theoretical and experimental/observational results on general-relativistic spin (rotation) effects in binary systems. A detailed discussion is given of the two-body Kepler problem and its first post-Newtonian generalization, including spin effects. Spin effects result from gravitational spin-orbit and spin-spin interactions (analogous to the corresponding case in quantum electrodynamics) and these effects are shown to manifest themselves in two ways: (a) precession of the spinning bodies per se and (b) precession of the orbit (which is further broke down into precessions of the argument of the periastron, the longitude of the ascending node and the inclination of the orbit). We also note the ambiguity that arises from use of the terminology frame-dragging, de Sitter precession and Lense-Thirring precession, in contrast to the unambiguous reference to spin-orbit and spin-spin precessions. Turning to one-body experiments, we discuss the recent results of the GP-B experiment, the Ciufolini-Pavlis Lageos experiment and lunar-laser ranging measurements (which actually involve three bodies). Two-body systems inevitably involve astronomical observations and we survey results obtained from the first binary pulsar system, a more recently discovered binary system and, finally, the highly significant discovery of a double-pulsar binary system.

## 1. – Introduction

This manuscript is, in essence, a progress report on general-relativistic spin effects in binary systems, which I lectured on in Varenna summer schools in 1974 [1] and 1975 [2]. Here, we use "spin" in the generic sense of meaning "internal spin" in the case of an elementary particle and "rotation" in the case of a macroscopic body. My 1974 lecture concentrated on the one-body problem [3] and analysis of the Schiff gyroscope experiment [4], now referred to as the Gravity Probe B (GP-B) experiment [5], which was launched on April 20, 2004 [6]. In addition, I discussed the possibility of C, P, and T effects occurring in the gravitational interaction. Not long after the 1974 summer school, a momentous event occurred: the discovery of a pulsar in a binary system by Hulse and Taylor [7] which bought into play a new array of possibilities for observing general relativistic effects. In particular, this motivated Barker and me to calculate two-body spin precession effects [8, 9], more details which were reported on later at the 1975 Varenna summer school. Thus, other than summarizing the key results from these lectures, I will concentrate here on discussing progress since then, in theory, experiment and observation. Section 2 is devoted to a general discussion of the Kepler problem and its first post-Newtonian generalization (order  $c^{-2}$ ) including spin effects. In section 3, we discuss a one-body system and related experiments and observations, not only the GP-B experiment but also the more recent results of Ciufolini and Pavlis [10] and the very early theoretical work of de Sitter [11, 12] and Lense-Thirring [13, 14] as well as the related work of Williams et al. [15] and Murphy et al. [16]. Section 4 concentrates on 2-body systems and we discuss, in particular, the observations of Stairs et al. [17] and the momentous discovery of a double binary system [18].

## 2. – The Kepler Problem and its Post-Newtonian Generalization

For a binary system, the Newtonian gravitational potential gives rise, for energy  $E < 0$ , to elliptic motion. We are interested in post-Newtonian generalizations (order  $c^{-2}$ ) arising from both spin and general relativistic effects. First, we present our notation [2]. Let  $m_1, \mathbf{r}_1, \mathbf{v}_1, \mathbf{P}_1, \mathbf{S}^{(1)}, \mathbf{n}^{(1)}, I^{(1)}$  and  $\boldsymbol{\omega}^{(1)}$  denote the mass, position, velocity, momentum, spin, unit vector in the spin direction, moment of inertia and angular velocity, respectively, of body 1. The same symbols, with  $1 \rightarrow 2$ , denote the corresponding quantities for body 2. In the center-of-mass system we have  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$  and  $\mathbf{P} = \mathbf{P}_1 = -\mathbf{P}_2 = \mu \mathbf{v}$  where  $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ . The reduced mass and the total mass are given by

$$(2.1) \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2}, \quad M \equiv m_1 + m_2.$$

Also, the orbital angular momentum is

$$(2.2) \quad \mathbf{L} = \mathbf{r} \times \mathbf{P} \equiv L \mathbf{n},$$

where  $\mathbf{n}$  is a unit vector in the  $\mathbf{L}$ -direction. In additon,

$$(2.3) \quad \frac{L/\mu}{a^2(1-e^2)^{\frac{1}{2}}} = \left(\frac{GM}{a^3}\right)^{\frac{1}{2}} = \frac{2\pi}{T} = \bar{\omega},$$

where  $T$  is the orbital period,  $e$  is the eccentricity,  $a$  is the semi-major axis, and  $\bar{\omega}$  is the average orbital angular velocity.

For a Newtonian elliptic orbit for two spherically symmetric bodies, the energy  $E$ , the orbital angular momentum  $\mathbf{L}$ , and the Runge-Lenz vector  $\mathbf{A}$  are constants of the motion, which can be written as

$$(2.4) \quad E/\mu = \frac{1}{2}v^2 - GM/r,$$

$$(2.5) \quad \mathbf{L}/\mu = \mathbf{r} \times \mathbf{v},$$

and

$$(2.6) \quad \mathbf{A}/\mu = \mathbf{v} \times (\mathbf{r} \times \mathbf{v}) - GM\mathbf{r}/r = \frac{1}{\mu}(\mathbf{v} \times \mathbf{L}) - GM\mathbf{r}/r,$$

where the direction of  $\mathbf{A}$  is along the major axis from the focus to the perihelion and is thus perpendicular to  $\mathbf{L}$ . However, when spin and post-Newtonian effects are taken into account (order  $c^{-2}$  beyond Newtonian theory) we found that the secular results for the precession of the orbit are

$$(2.7) \quad \dot{E}_{av} = 0,$$

$$(2.8) \quad \dot{\mathbf{L}}_{av} = \boldsymbol{\Omega}^* \times \mathbf{L},$$

$$(2.9) \quad \dot{\mathbf{A}}_{av} = \boldsymbol{\Omega}^* \times \mathbf{A},$$

where

$$(2.10) \quad \boldsymbol{\Omega}^* = \boldsymbol{\Omega}^{*(E)} + \boldsymbol{\Omega}^{*(1)} + \boldsymbol{\Omega}^{*(2)} + \boldsymbol{\Omega}^{*(1,2)},$$

consists of contributions from a spin-independent term  $\Omega^{*(E)}$  (Einstein's precession of the perihelion), in addition to  $\mathbf{S}^{(1)}$ ,  $\mathbf{S}^{(2)}$  and  $\mathbf{S}^{(1)}\mathbf{S}^{(2)}$  terms, respectively, which are all given explicitly in [8]. We note that, on the average,  $\mathbf{L}$  and  $\mathbf{A}$  precess at the same rates, so that the Kepler ellipse as a whole precesses with the angular velocity  $\Omega^*$ . It was also found convenient (in order to adhere to astronomical and space physics practise) to write  $\Omega^*$  in the form

$$(2.11) \quad \Omega^* = \frac{d\Omega}{dt}\mathbf{n}_0 + \frac{d\omega}{dt}\mathbf{n} + \frac{di}{dt} \frac{\mathbf{n}_0 \times \mathbf{n}}{|\mathbf{n}_0 \times \mathbf{n}|},$$

where  $\Omega, \omega$  and  $i$  denote the longitude of the ascending node, the argument of the periastron and the inclination of the orbit, respectively, in the reference system of the plane of the sky (the tangent plane to the celestial sphere at the center of mass of the binary system) [2, 3]. In addition,  $\mathbf{n}_0$  is a unit vector normal to the plane of the sky directed from the center of mass of the binary system towards the Earth. The angle between  $\mathbf{n}_0$  and  $\mathbf{n}$  is the inclination  $i$ . In the absence of spin, only the periastron precession is present.

In addition, each spin precesses at rates  $\Omega^{(1)}$  and  $\Omega^{(2)}$ , which when averaged over an orbital period we write as

$$(2.12) \quad \dot{\mathbf{S}}_{av}^{(1)} = \Omega_{av}^{(1)} \times \mathbf{S}^{(1)},$$

and similarly for  $1 \rightarrow 2$ . However, as we recently stressed "gravitational effects due to rotation (spin) are best described, using the language of QED, as spin-orbit and spin-spin effects since they also denote the interactions by which such effects are measured; in fact these are the only such spin contributions to the basic Hamiltonian describing the gravitational two-body system with arbitrary masses, spins and quadrupole moments. They manifest themselves in just two ways, spin and orbital precessions, and whereas these can be measured in a variety of ways (for example, as discussed above, orbital precession can be subdivided into periastron, nodal and inclination precessions) such different measurements are simply 'variations on the theme'" [19]. In fact, in the basic Hamiltonian for the problem [8], the spin-orbit terms are of the form  $\mathbf{S}^{(1)} \cdot \mathbf{L}$  and  $\mathbf{S}^{(2)} \cdot \mathbf{L}$  whereas the spin-spin terms are of the form  $\{[3(\mathbf{S}^{(1)} \cdot \mathbf{r})(\mathbf{S}^{(2)} \cdot \mathbf{r})/r^2 - \mathbf{S}^{(1)} \cdot \mathbf{S}^{(2)}]\}$ , which is completely analogous to what occurs in QED, as we pointed out in [1]. Thus, we were able to conclude [1] that all such effects in QED have their analogy in general relativity and, except for (important) numerical factors, the latter results may be obtained from the former by simply letting  $e^2 \rightarrow Gm_1m_2$ . Hence it is convenient to write (with a similar equation for  $1 \rightarrow 2$ )

$$(2.13) \quad \begin{aligned} \Omega_{av}^{(1)} &= \Omega_{so}^{(1)} + \Omega_{ss}^{(1)} \\ &\equiv \Omega_{ds}^{(1)} + \Omega_{LT}^{(1)}, \end{aligned}$$

where the second equality is often written to make contact with common usage,  $ds$  denoting de Sitter [11, 12] and  $LT$  denoting Lense-Thirring [13, 14]. The former is also referred to as the geodetic effect and the latter as a "frame-dragging" effect. However, as we pointed out in [19], the de Sitter precession can also be regarded as a frame-dragging effect, which is even more obvious in the case of two-body systems (but which is also readily seen in the case of the GP-B experiment if one considers an observer in the frame of the gyroscope). Further confusion associated with the terminology "Lense-Thirring effect" arises from the fact that it is also used to discuss earth spin effects on orbital motion, as we discuss in more length below (subsect. 3.3). Thus, to avoid confusion, we feel that it is best to just use the terminology given in the first equality of (2.13). We also note that  $\mathbf{\Omega}_{so}^{(1)}$  and  $\mathbf{\Omega}_{so}^{(2)}$  (explicit results for which are given in [8]) do not depend on the spins and always point along  $\vec{L}$  whereas  $\mathbf{\Omega}_{ss}^{(1)}$  depends on the spin of body 2 and points along  $\{[\mathbf{n}^{(2)} - 3(\mathbf{n} \cdot \mathbf{n}^{(2)})\mathbf{n}]\}$ .

To conclude this section, we make some general remarks. First, we note that since gravitational radiation only arises at order  $c^{-5}$ , and defining the total angular momentum

$$(2.14) \quad \mathbf{J} \equiv \mathbf{L} + \mathbf{S}^{(1)} + \mathbf{S}^{(2)},$$

we obtained [8]

$$(2.15) \quad \frac{d\mathbf{J}}{dt} = 0.$$

again underlining the inter-relationship between the spin and orbital precessions. Secondly, we note that the largest precession is always the periastron precession and it is the only one that remains in the absence of spin. For a particular system, the magnitude of the periastron precession and the spin-orbit precessions depends significantly on the value of the product of the orbital angular velocity  $\bar{\omega}$  and the gravitational coupling constant

$$(2.16) \quad \alpha_g = (GM/c^2 a).$$

where  $a$  is the semi-major axis of the Keplerian ellipse. Thus, for the GP-B satellite orbiting the earth,  $\bar{\omega} = 1.074 \times 10^{-3} s^{-1}$  and  $\alpha_g = 6.958 \times 10^{-10}$ , so that  $\alpha_g \bar{\omega} = 7.473 \times 10^{-13} s^{-1} = (1.35 \times 10^{-4})^0 / yr$ . On the other hand, for the double binary system,  $\bar{\omega} = 7.112 \times 10^{-4} s^{-1}$  and  $\alpha_g = 4.348 \times 10^{-6}$  so that  $\alpha_g \bar{\omega} = 3.092 \times 10^{-9} s^{-1} = 5^\circ.591 / yr$ . As we will see below, this difference, amounting to 4 orders of magnitude, is reflected in the significantly larger precessions (both orbital and spin) obtained for binary star systems compared to earth related systems. For example, this order-of-magnitude calculation is borne out by an exact calculation of the spin-orbit precession, which gives (in degrees per year)  $1.84 \times 10^{-3}$  for the GP-B gyroscope and 4.8 and 5.1 for pulsars A and B, respectively, which constitute the double binary system, to be discussed below. Thirdly,

the inter-relationship between the spin and orbital precessions, as expressed in (2.15), may be seen explicitly by writing our exact basic results [8] in the following succinct form (recalling again that the precession rates are averaged over an orbital period)

$$\begin{aligned}
 \frac{d\mathbf{S}^{(1)}}{dt} &= \left( \boldsymbol{\Omega}_{so}^{(1)} + \boldsymbol{\Omega}_{ss}^{(1)} \right) \times \mathbf{S}^{(1)} \\
 &= \frac{3}{2(1-e^2)} \alpha_g \bar{\omega} \frac{m_2 + \mu/3}{M} \left( \mathbf{n} \times \mathbf{S}^{(1)} \right) \\
 (2.17) \quad &+ \frac{1}{2(1-e^2)} \alpha_g \frac{S^{(2)}}{Ma^2} \left[ \left( \mathbf{n}^{(2)} \times \mathbf{S}^{(1)} \right) - 3 \left( \mathbf{n} \cdot \mathbf{n}^{(2)} \right) \left( \mathbf{n} \times \mathbf{S}^{(1)} \right) \right],
 \end{aligned}$$

$$\begin{aligned}
 \frac{d\mathbf{S}^{(2)}}{dt} &= \left( \boldsymbol{\Omega}_{so}^{(2)} + \boldsymbol{\Omega}_{ss}^{(2)} \right) \times \mathbf{S}^{(2)} \\
 &= \frac{3}{2(1-e^2)} \alpha_g \bar{\omega} \frac{m_1 + \mu/3}{M} \left( \mathbf{n} \times \mathbf{S}^{(2)} \right) \\
 (2.18) \quad &+ \frac{1}{2(1-e^2)} \alpha_g \frac{S^{(1)}}{Ma^2} \left[ \left( \mathbf{n}^{(1)} \times \mathbf{S}^{(2)} \right) - 3 \left( \mathbf{n} \cdot \mathbf{n}^{(1)} \right) \left( \mathbf{n} \times \mathbf{S}^{(2)} \right) \right],
 \end{aligned}$$

and

$$\begin{aligned}
 \frac{d\mathbf{L}}{dt} &= \left( \boldsymbol{\Omega}^{*(E)} + \boldsymbol{\Omega}^{*(1)} + \boldsymbol{\Omega}^{*(2)} + \boldsymbol{\Omega}^{(1,2)} \right) \times \mathbf{L} \\
 &= - \left\{ \left( \boldsymbol{\Omega}_{so}^{(1)} \times \mathbf{S}^{(1)} \right) + \left( \boldsymbol{\Omega}_{so}^{(2)} \times \mathbf{S}_{so}^{(2)} \right) + \left[ \left( \boldsymbol{\Omega}_{ss}^{(1)} \times \mathbf{S}^{(1)} \right) + \left( \boldsymbol{\Omega}_{ss}^{(2)} \times \mathbf{S}^{(2)} \right) \right] \right\} \\
 (2.19) \quad &= - \left( \frac{d\mathbf{S}^{(1)}}{dt} + \frac{d\mathbf{S}^{(2)}}{dt} \right),
 \end{aligned}$$

which is consistent with (2.14) and (2.15). This is another manifestation of the fact that there are simply two basic gravito-magnetic interactions in general relativity, spin-orbit and spin-spin. We note that  $\boldsymbol{\Omega}^{*(E)}$  does not contribute to  $\frac{d\mathbf{L}}{dt}$  whereas for  $\frac{d\mathbf{A}}{dt}$  it is the dominant Einstein-Robertson periastron precession term. It is also immediately clear from these results that the various precession rates are proportional to  $\alpha_g$  times the average orbital rate, consistent with the fact that we are working to order  $c^{-2}$  (first post-Newtonian approximation). After this general discourse, we now turn to a discussion of various experiments to detect spin-orbit and spin-spin effects in gravitational theory.

### 3. – One-Body System

**3.1. The GP-B experiment.** – As already mentioned, it was launched on April 20, 2004. Data collection covered the period Aug. 2004-Sept. 2005, when the He in the

dewar had completely leaked out but, due to unexpected "–measurement uncertainty–," the announcement of final results has been delayed until December 2007.

Here  $m_2$  (earth mass)  $\gg m_1$  (gyroscope mass) and  $\mathbf{S}^{(2)}$  is along the earth's rotation axis. Since  $\boldsymbol{\Omega}^{(1)}$  is always along  $\mathbf{L}$ , it turns out that a polar orbit results in  $\boldsymbol{\Omega}_{so}^{(1)}$  and  $\boldsymbol{\Omega}_{ss}^{(1)}$  being at right angles resulting, in principle, in a clear separation of the two effects. However, due to a variety of other effects [20, 21], the analysis turns out to be much more complicated. The calculated precessional rates for the GP-B experiment, in units of arc-seconds per year ( $2.78 \times 10^{-4}$  degrees per year) are 6.61 for  $\boldsymbol{\Omega}_{so}^{(1)}$  and  $4.1 \times 10^{-2}$  for  $\boldsymbol{\Omega}_{ss}^{(2)}$  and the hope is that the accuracy achieved is better than  $5 \times 10^{-4}$  ( $1.4 \times 10^{-7}$  degrees) per year [22]. However, whether this is achievable in view of the problems encountered [6, 22, 23] remains to be seen. Among the concerns is the polhode motion arising from the fact that the gyroscopes are not perfect sphere. Thus, not only can a quadrupole moment arise from the manufacturing process but, in addition, even an ideal perfectly spherical gyro acquires a quadrupole moment due to its rotation [20]. Using the quoted values of the gyro diameter and average spin rate [22] we obtain a value for  $(\Delta I/I)$ , arising from this effect, of  $7.1 \times 10^{-6}$ , which is comparable to that arising from the manufacturing process. An additional problem as far as the experiment is concerned is that the two different quadrupole moments are located at an unknown angle from each other, aggravated by the fact that "–there was no way to know about which axis each gyro (of the four total) was initially spun up–" [23]. Moreover, the "–wide variation in spin-down rates–" will be reflected by a time variation in the value of  $(\Delta I/I)$  associated with rotation (since  $\Delta I$  is proportional to the square of the gyro's angular velocity), which leads to a time variation in the polhode periods. In addition, we note that the two different quadrupole moments also affect the orbit equations resulting in a concomitant contribution to the spin precession rate [20]. The inter-connection of these various parameters presents a challenge to the interpretation of the data, particularly for the determination of  $\boldsymbol{\Omega}_{ss}$ , which is 161 times smaller than  $\boldsymbol{\Omega}_{so}$ . The data analysis is further aggravated [22, 23] by the fact that unexpected classical torques are produced on the gyros, together with damping of their polhode motion, which are caused by interactions between the gyro rotors and their housings due to electrostatic patches (potential differences).

Since nearly five decades has passed since the GP-B experiment was conceived and designed, there has been much progress in achieving the same results by a variety of other methods which we will discuss below. In particular, there are now three independent determinations of  $\boldsymbol{\Omega}_{so}$  consistent with the predicted results arising from Einstein's theory and, in fact, one of them pertains to a 2-body system for which additional interesting features arise. Also, whereas it is reported that GP-B has confirmed the predicted spin-orbit result to better than 1% [22], Murphy et al. [16] are claiming 10 times greater accuracy based on lunar laser ranging. Thus, more attention will be focussed on the main "selling-point" of the experiment, that is the determination of  $\boldsymbol{\Omega}_{ss}$ . But, whether this is achievable is very problematic in view of the polhode problems already mentioned and other various classical torques on the gyroscopes [22]. Of course, there have been many significant "spin-offs" connected with the experiment. But, whether or not this will be regarded as justification for the time spent and the cost (over \$760 million) and in view

of the fact that it required the U.S. Congress to keep it alive after NASA "has tried to cancel the mission at least three times" [24, 25] will surely give rise to an analysis of the sociology and politics of science research and funding, along the lines that Collins presented for the search for gravitational waves [26].

**3.2. Determination of  $\Omega_{so}$  from lunar-laser-ranging measurements.** – As already alluded to above and discussed in detail in [19], the de Sitter precession  $\Omega_{so}(\Omega_{ds})$  is also a frame-dragging effect and "provides an accurate benchmark measurement of spin-orbit effects which GPB needs to emulate." [19]. In essence, starting with the work of de Sitter and investigated in detail by many groups [27, 15, 16], it is clear that "because of its distance from the sun, the earth-moon system can be regarded as a single body which is rotating in the gravitational field of the sun. In other words, the earth-moon system is essentially a gyroscope in the field of the sun and its frame-dragging effect due to interaction with the sun has been measured, using lunar laser ranging" [19]. First we note that the relevant  $\alpha_g$  is  $9.8724 \times 10^{-9}$  and  $\bar{\omega} = 1.991 \times 10^{-7} s^{-1}$ . The product  $\frac{3}{2}\alpha_g\bar{\omega}$  results in a de Sitter [11, 12] spin-orbit contribution which is reflected in a contribution to the lunar perigee  $\approx 19.2$  m-sec/yr [27]. Second, and more recently, Murphy et al. [16] pointed out that the current accuracy of lunar laser ranging is such that it provides a test of spin-orbit coupling to "approximately 0.1% accuracy - better than the anticipated accuracy of the gravity - Probe B result" and that "a new effort in LLR is poised to deliver order-of-magnitude improvements in range precision".

**3.3. Ciufolini's LAGEOS experiment.** – The fundamental idea for this work goes back to Ciufolini's work in mid 1980, details of which may be found in [28]. In their recent paper [10, 29], Ciufolini and Pavlis used in their analysis two satellites, LAGEOS (launched in 1976) and LAGEOS2 (launched in 1992) which simply consist of a heavy sphere of retro-reflectors which reflect short laser pulses sent from earth, leading to a determination of earth-satellite distances with a precision of a few *mm*. The focus is on the orbital motion of the satellites and specifically a determination of  $d\Omega/dt$ , the nodal precession, due to the spin of the earth. As already emphasized, this is a spin-orbit effect, involving the spin of the earth and the angular momentum of the orbit; the only difference with the corresponding spin-orbit measurement in the GP-B experiment is that the spin under discussion there is the spin of the gyroscope (the spin of the earth coming into play only in the spin-spin interaction involving the earth spin and the gyroscope spin). Our emphasis on precise terminology stems from the fact that there is confusion associated with the use of the terminology "Lense-Thirring effect"; some authors, such as [10], use it when the earth's spin is playing a role whereas others, such as [5, 22] use it to denote the interaction between the earth's spin and the spin of the gyroscope. The emphasis in [10] on nodal motion is based on the fact that, in this case, non-gravitational perturbations are easiest to handle. Also, the use of two satellites made it possible to eliminate the effect of the earth's quadrupole moment. The use of the GRACE earth gravity model [29], resulting in very small uncertainties arising from other harmonics, led to a value of  $48.2 mas yr^{-1}$  for the nodal precession, accurate to a precision of 10%. This is a very impressive result



and the error estimate is bound to become even smaller as better results emerge for the various multipoles of the earth's field and for the expected improvements in laser-ranging precision. For an excellent review of recent measurements of frame dragging using earth-orbiting satellites, we refer to Ciufolini's Nature review [44], where he remarks that the laser-ranged Italian satellite LARES should " - - in future provide an improved test of the Earth's gravitomagnetism with accuracy of the order of 1%".

#### 4. – Two-Body Systems

**4.1. The binary pulsar PSR 1913+16 [7].** – It was the first discovery of a pulsar in a binary system and stimulated Barker and me to extend our one-body analysis [3] to the two-body arena [8], the results of which were immediately applied to the Hulse-Taylor system [9].

In the two-body system, new and unexpected results were obtained. The only previous two-body calculation was that of Robertson [31] who neglected spin and considered only two test particles for which he calculated the periastron precession, the result for which was the same as the one-body calculation except for the replacement  $m_2 \rightarrow m_1 + m_2 \equiv M$ . However, when spin is taken into account, the situation is very different. What we found is that, for the spin precession of body 1, the replacement in the formula for  $\Omega_{so}^{(1)}$  is  $m_2 \rightarrow m_2 + (\mu/3)$  and, in the case of  $\Omega_{so}^{(2)}$ , the replacement is  $m_1 \rightarrow m_1 + (\mu/3)$ , where we recall, from (2.1), that  $\mu$  is the reduced mass. Similar replacements occur for the orbital precessions  $\Omega^{*(1)}$  and  $\Omega^{*(2)}$ , associated with the spin-orbit interaction contributions from the spins of bodies 1 and 2, respectively. This is shown explicitly in (2.17) to (2.19). Here  $\alpha_g = 2 \times 10^{-6}$ , compared to a value of  $7 \times 10^{-10}$  for the earth-gyro system and thus we expect correspondingly large numbers for the various precession angles. In line with these expectations, the observations give  $\Omega^{*(E)} = 4^0.2/yr$ , in agreement with theory. The theoretical value for  $\Omega_{so}^{(1)}$  is  $1^0.1/yr$  (where  $m_1 = 1.42m_\odot$  refers to the spinning pulsar) but this quantity is difficult to deduce from the data. Two obvious ways suggest themselves: (a) wait for the pulsar beam (which is directed in a narrow cone toward the earth) to go outside the line of sight, (which could take a long time since the precessional period is about 327 years), or (b) observe the changes over time in the shape of the pulses, which have now been seen [32, 33] and which give qualitative agreement with theory but due to uncertainties in the beam shape, a quantitative comparison has not yet been possible.

**4.2. The binary pulsar PSR B1534+12 [17].** – Here, based on the two-body spin precession formula given in [8] and [9], the theoretical precession rate is  $0.51^0/yr$ . Observations amounting to over 400 hours led to a determination of the time evolution of (a) the angle between the spin and the magnetic pole and (b) the minimum impact angle of the magnetic pole on the line of sight, from which the authors deduced direct evidence of geodetic (spin-orbit) precession. This was bolstered by an independent determination based on time variation of the shape of the profile of the pulses. The conclusion yielded a "–measurement of the precession time consistent with the predictions of general

relativity—” [17] and the authors note that “—although the precision is, as yet, limited—”, future plans should allow for improvement.

**4.3. The double binary system PSR J0737-3039 [18, 34].** — This is a highly significant discovery, due to the fact that it is the only binary system discovered in which both neutron stars are radio pulses; in particular it widens the scope for tests of general relativity [35] and for the study of the magnetospheres surrounding pulsars. This binary has an orbital period of 2.4 hours and the periastron precession was observed to be  $16^\circ.9/\text{yr}$ . Pulsar A has a period of  $22.7\text{ms}$  whereas pulsar B has a period of  $2.77\text{s}$ , that is A spins 122 times faster. A detailed listing of measured and derived parameters are listed in [18, 34, 35]; in particular, in units of a solar mass,  $M = 2.58708$  with estimated values of  $m_A = 1.3381$  and  $m_B = 1.2489$ . This leads to  $\alpha_g \approx 4.4 \times 10^{-6}$ . In addition, the corresponding reduced mass  $\mu$  is 0.64596. Thus, using the two-body spin precession formula [8, 9], given in (2.17) and (2.18), the expected spin-orbit spin precession rates are  $4^\circ.8/\text{yr}$  for pulsar A and  $5^\circ.1/\text{yr}$  for pulsar B (which corresponds to geodetic precession periods of 75 years for A and 71 years for B). Since the corresponding precession rate for the GP-B earth is  $1.838 \times 10^{-3}$  degrees per year, we see that the number for pulsar A is larger by a factor of  $2.7 \times 10^3$ . Despite these large values, they have not yet been observationally verified. However, it may be possible to deduce the effect of the two-body spin-orbit term from the fact that it also results in a value of  $\approx 4.06$  arc-sec/yr for the rate of precession of both the angular momentum and the Runge-Lenz vector of the orbit about the pulsar spin direction [36]. This is reflected in a precession of the inclination of the orbit [36] and there is also a contribution to the perihelion precession. However, in the former case, such an observation is not helped by the fact that the orbital plane is close to the line of sight, the angle of inclination being  $88^\circ.69$  [35]. The latter case is thus probably more promising since the perihelion precession has been measured to a relative precision  $\approx 10^{-5}$ . We expect that the second-order spin-independent post-Newtonian contribution to the periastron precession to be smaller than the lowest-order Einstein-Robertson contribution by a factor  $\approx \alpha_g = 4 \times 10^{-6}$ . On the other hand, the lowest-order spin-orbit contribution of the fast pulsar A will be smaller by a factor,  $F$  say, where

$$(4.1) \quad F = \frac{\omega^{(1)}}{\bar{\omega}} \left( \frac{R}{a} \right)^2,$$

where  $\omega^{(1)} = 44.0545\text{s}^{-1}$  is the precession frequency of pulsar A [35] and where  $R \approx 15\text{kms}$  is the radius of the pulsar A neutron star. Thus, since  $(\omega^{(1)}/\bar{\omega}) = 3.89 \times 10^3$ , and  $(R/a) = 1.71 \times 10^{-5}$ , we see that  $F = 1.1 \times 10^{-4}$ . Thus, the lowest-order spin-orbit contribution could be  $\approx 10$  times the second-order spin-independent contribution and also the relative precision  $\approx 10^{-5}$ . This translates to a contribution to the periastron precession  $\approx (16.9 \times 10^{-4})^\circ/\text{yr} \approx 5$  arc-sec/yr, keeping in mind the uncertainties underlying this calculation. However, if the observations provide even better relative precision

in the future, they could ”–provide a way to obtain accurate information of the moment of inertia of neutron stars–.” [36].

## 5. – Conclusions

On the theoretical side, the general relativistic Hamiltonian for a two-body system with spin, to order  $c^{-2}$ , contains spin-orbit and spin-spin contributions, analogous to the corresponding Hamiltonian for QED. These terms separately manifest themselves as spin and orbital precessions, which are in fact intimately related, reflecting their common origin. Also, it was convenient from an observational point of view, to separate the orbital precession into periastron, nodal and inclination angle precessions. Thus, a measurement of any of these precessions is potentially a test of gravitomagnetism (spin-orbit and spin-spin effects).

As we discussed above, there is already strong evidence (subsects. 3.2, 3.3 and 4.2) for the predicted spin-orbit effects in general relativity. This bears out the conclusion of a NASA 2003 scientific review who ”–found ”some erosion” in the scientific value of the frame-dragging experiment–” [37]. However, it does appear, from the initial announcement of specific results from the GP-B experiment that an accuracy better than 1% may have been achieved for the geodetic effect but the lunar laser experiment is already much better, achieving an accuracy of 0.1%. Also, the other experiments, as distinct from GP-B, have the advantage of being on-going experiments. For readers interested in other theories of gravitation (for which I believe there is neither strong theoretical, experimental or observational evidence), a detailed discussion is given in [38]. In particular, it is clear that in all these other theories, the intimate relationship between the spin-orbit measurements of the GPB group and those of Ciufolini and Pavlis [10] still holds.

We emphasize that our considerations were confined to order  $c^{-2}$ , which is an excellent approximation for the systems considered for which the largest value of  $\alpha_g$  is  $4.4 \times 10^{-6}$ . However, motivated by the search for gravitational radiation, there is widespread activity in investigating the final evolution of a binary spinning black-hole system; the results given in [8] provide a touchstone for sufficiently large separations but strong field effects soon play a dominant role as the merger takes place so that massive computational effects are necessary to extract meaningful results [39].

Finally, we note that the theoretical work underpinning the results presented here are treated in depth in a companion paper [40]. In particular, we discuss relevant conceptual matters dealing with the relation between velocity and momentum (especially for particles with spin), the non-uniqueness of spin supplementary conditions and the choice of coordinates even at the classical special relativistic level, the fact that a spinning particle necessarily has a minimum radius, the corresponding concepts in quantum theory (relating to such topics as the Foldy-Wouthuysen transformation and position operators) and the fact that the spin effects in quantum electrodynamics (obtained from one-photon exchange) have their analogy in general relativity (obtained from one-graviton exchange or purely classical calculations) to the extent that except for (important) numerical factors,

the latter results may be obtained from the former by simply letting  $e^2 \rightarrow Gm_1m_2$ .

## 6. – Post-Script

A recent Comment by Kopeiken [41] argues that " - - lunar laser ranging (LLR) is not currently capable to detect gravitomagnetic effects - - - ", based on the fact that a Newtonian-like translation from the solar system barycentric (SSB) frame to the geocentric frame (that is the frame co-moving with the earth) gives rise to additional gravitomagnetic terms which " - - makes LLR insensitive to the gravitomagnetic interaction". As an example of such extra contributions, we note that coordinate transformations give rise to Thomas precession contributions of the same order of magnitude as those arising from post-Newtonian effects [42, 28]. However, for the 3-body earth-moon-sun system, there are several velocity-dependent contributions and a Reply by Murphy et. al. [43] to the Kopeiken Comment disputes his conclusion on the basis that " - - - transformations of the velocity-dependent terms from one frame to another - - - [are] strongly constrained by experiment".

More recently, Ciufolini in a lucid analysis [44], reiterates a point which he made earlier in his book with Wheeler [28] that " - - - the de Sitter effect and the Lense-Thirring drag are fundamentally different phenomena - - - " [28] since the curvature invariant  $*R.R$  is zero in the former case (which is generated by the translational motion of a mass) but non-zero in the latter case (which is generated by the rotation of a mass). Ciufolini then goes on to stress that " - - - one cannot derive rotational gravitomagnetic effects from translational ones, unless making additional theoretical hypotheses such as the linear superposition of the gravitomagnetic effects." While we agree with Ciufolini, from our point of view, both of the effects under discussion arise from a common spin-orbit term in the basic Hamiltonian  $H$ , which can manifest itself in various ways. In particular, as detailed above, the same spin-orbit term gives rise to both a precession of the spin (or rotation axis) per se and also a precession of the perihelion of the orbit. In particular, the rate of precession of the spin arising from the spin-orbit term in  $H$  depends on the angular momentum and not on the spin whereas the rate of orbital precession depends on the spin but not on the angular momentum. The rate of precession of the GP-B gyroscope orbiting the earth is representative of the former case and so is the earth-moon gyroscope orbiting the sun. Concomitantly, in both cases, there is an associated orbital precession; this is negligibly small in the case of the GP-B gyro but not so for the case of the earth-moon gyroscope because the latter is itself a two-body system forming part of a three-body system. As a consequence, the motions of the earth and the moon can be separately tracked leading, in particular, to spin-orbit amplitude contributions to the lunar orbit at the 6 m level [16].

We should also emphasize that, as already discussed, there are two separate spin-orbit contributions. However, in all one-body problems, the dominant spin-orbit contribution to the spin precession is associated with the spin of the smaller mass [see (2.17) and (2.18)] i.e. the gyro mass in the case of GP-B and the earth-moon mass in the case of LLR. Of course, there is also a spin-spin contribution in both cases; an attempt

has been made to measure this in the case of GP-B but it is generally negligible for contributions to one-body orbital precessions (compared to spin-orbit contributions and the dominant Einstein-Robertson periastron precession term). On the other hand, for two-body systems, such as the double binary system, both spin-orbit contributions to spin precession are important and, as already noted, in Section 4.3, their contribution to the periastron precession of the orbit of the double binary system may also be measurable in further observations.

More generally, we emphasize that our analyses and calculations demonstrate the universality of gravitational spin effects regardless of whether the source is the spin of an elementary particle, the rotation of a macroscopic body or a translational effect. I thank Professor Herbert Pfister for comments which initiated this post-script. As a further update, we note that our result for spin-orbit precession in a 2-body system [8] has been verified, to an accuracy of 13%, by Breton et al., who observed a precession of the spin axis of pulsar B in the double binary system by an amount  $4.77^\circ/\text{yr}$ . [45].

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